Graphing Polynomials (Part 2)

These notes are intended as a summary of section 1.4 (p. 37 – 45) in your workbook. You should also read the section for more complete explanations and additional examples.

The graphs of polynomial functions are generally drawn using one of two methods:

1. Using the properties of the function (degree, zeros, etc.).

    - this method is used when the polynomial can be factored

2. Using a table of values.

    - this method is used when the polynomial cannot be factored

Graphing a Polynomial by its Properties

In order to graph a polynomial using its properties, follow the steps below:

1. Plot the zeros and the y-intercept.

    - factor the function to determine the zeros

    - the y-intercept is *always* equal to the constant term

2. Determine which way the ends of the graph point.

    - this is done using the **leading coefficient test** (see below)

3. Determine if the graph lies above or below the x-axis between each pair of zeros.

    - this is done by choosing any x-value between the zeros and plugging it into the function

4. Plot the graph.

    - draw a smooth, continuous curve that passes through all points

    - make sure the graph is above or below the x-axis in the correct intervals (as determined in step 3)

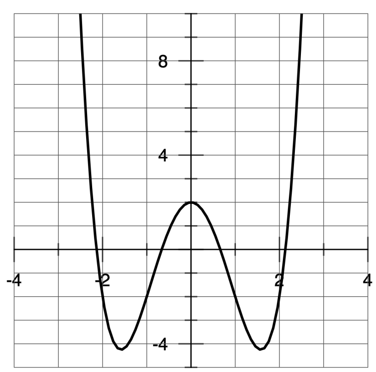
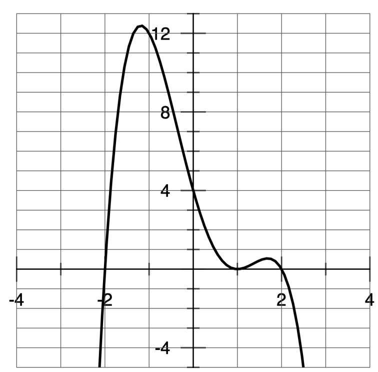
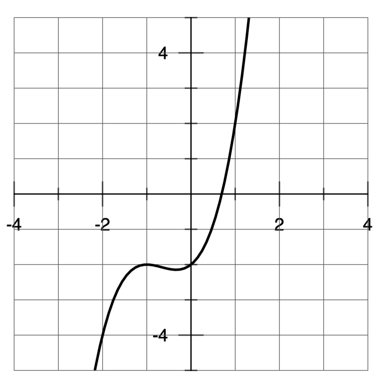
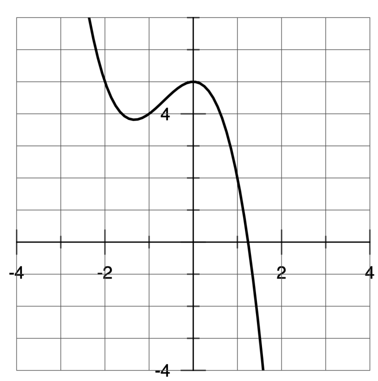
    - **Note**: the **multiplicity** of each zero determines what happens when the graph intersects the x-axis (see below for more details)

Leading Coefficient Test

By examining the degree of a polynomial, and the sign of its leading coefficient, you can determine the end behavior of the graph.

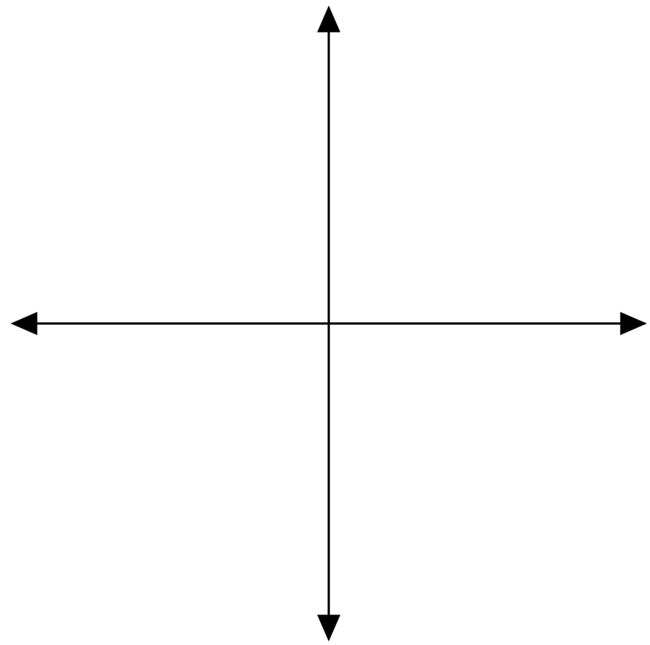
	Leading Coefficient	
Degree	**Positive**	**Negative**
Even	both ends point up	both ends point down
Odd	right end points up left end points down	right end points down left end points up

The table below has some examples to illustrate this test.

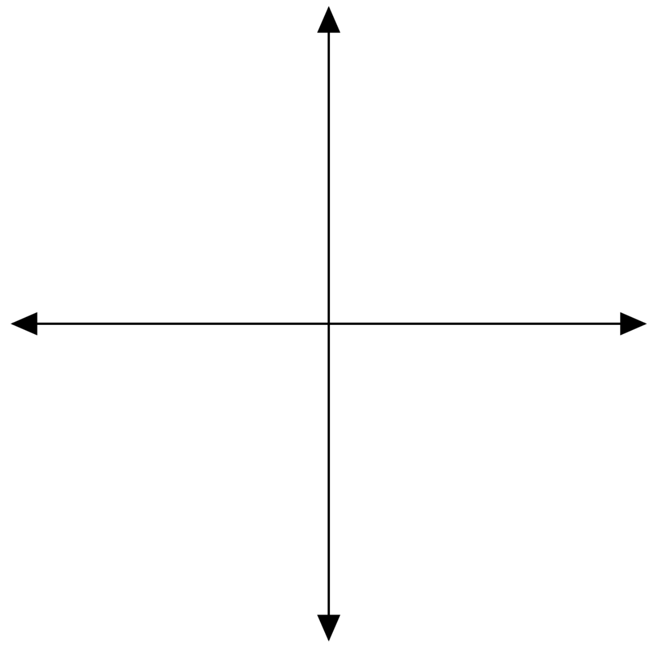
<table>
<tr><td></td><td colspan="2">Leading Coefficient</td></tr>
<tr><td>Degree</td><td>Positive</td><td>Negative</td></tr>
<tr><td>Even</td><td>$f\left(x\right)=x^4-5x^2+2$
</td><td>$f\left(x\right)=-x^4+2x^3+3x^2-8x+4$
</td></tr>
<tr><td>Odd</td><td>$f\left(x\right)=x^3+2x^2+x-2$
</td><td>$f\left(x\right)=-x^3-2x^2+5$
</td></tr>
</table>

Example (not in workbook)

Sketch the graph of $f\left(x\right)=x^3-2x^2-11x+12$ (zeros are –3, 1, and 4).



Example 2 (sidebar p. 42)
Sketch the graph of the polynomial function: $f(x) = x^3 + x^2 - 6x$.



Multiplicity

A polynomial function may have a repeated zero. For example, the equation $x^2 - 2x + 1 = 0$ can be factored to

$$(x-1)(x-1)=0 \qquad \text{or} \qquad (x-1)^2 = 0$$

Since 1 is a zero of the equation twice, we say it is a zero with **multiplicity** 2. (Note: the exponent on the factor is the same as the multiplicity).

The multiplicity of a zero affects the appearance of the graph as follows:

1. If the multiplicity of a zero is 1, then the graph passes straight through that zero.

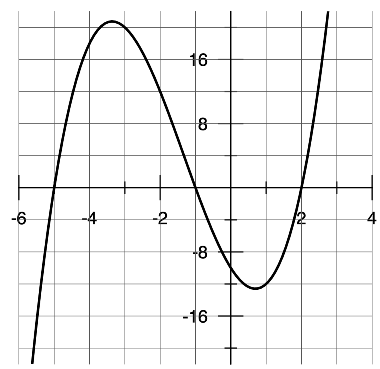
2. If the multiplicity of a zero is odd (but not 1), then the graph passes through the zero, but does so **tangent to the *x*-axis**.

3. If the multiplicity of a zero is even, then the graph does not pass through the zero. Instead, it "bounces" off of the x-axis at that zero.

Here are some examples to illustrate this concept.

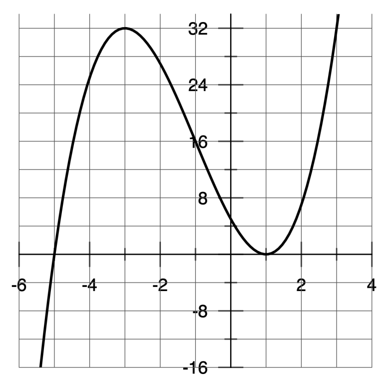
$f(x) = x^3 + 4x^2 - 7x - 10$ has zeros –1, 2, and –5 (all have multiplicity one). Its graph is shown to the right.

Note that it passes straight through all three zeros.

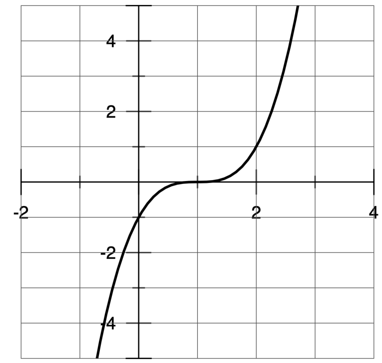


$f(x) = x^3 + 3x^2 - 9x + 5$ has zeros 1, 1, and –5 (1 is a zero with multiplicity two, while –5 is a zero with multiplicity one). Its graph is shown to the right.

Note that it passes straight through the zero at –5, but it "bounces" off of the axis at the zero 1 (with even multiplicity).



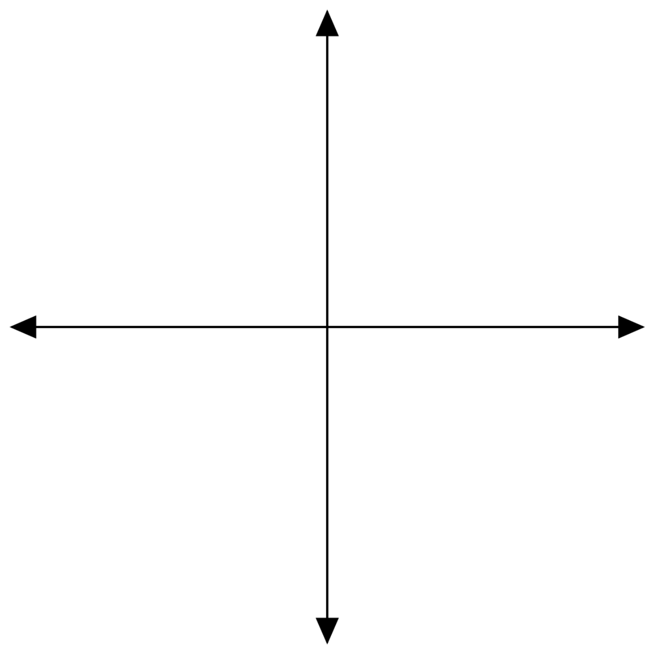
$f(x) = x^3 - 3x^2 + 3x - 1$ has zeros 1, 1, and 1 (i.e. 1 is a zero with multiplicity three). Its graph is shown to the right.



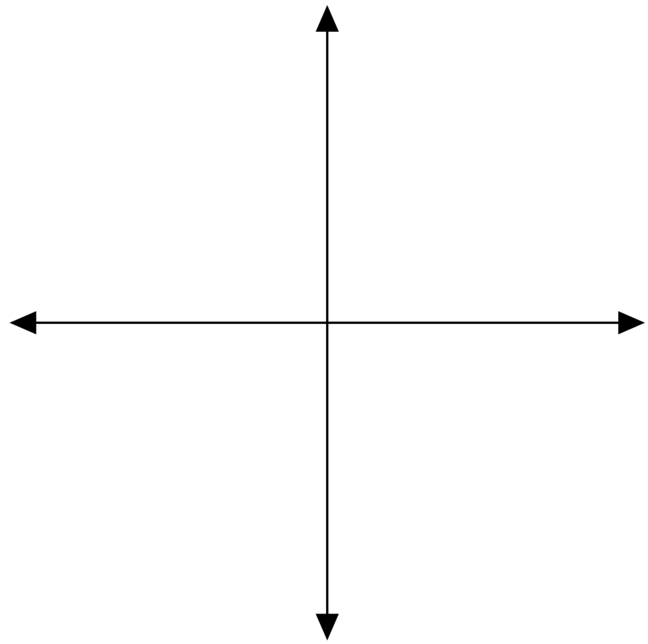
Note that it passes through the zero at 1, but it doesn't pass straight through. Instead, it "levels off" and passes through the point parallel to the *x*-axis.

Example 3 (sidebar p. 45)
Sketch the graph of each polynomial function.

a) $f(x) = (x+1)^4 (x-2)$

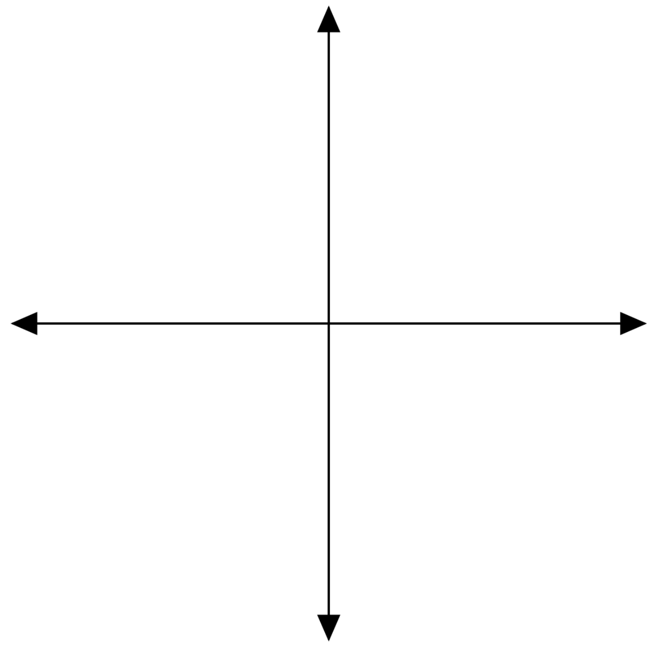


b) $g(x) = -(x+1)^3(x-3)$



Example (not in workbook)

Sketch the graph of $f(x)=x^4-6x^3-3x^2+56x-48$.



Homework: #5, 7, 9 – 13 in the exercises (p. 46 – 54). Answers on p. 55.